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How do we know that the creations of worlds are not determined by falling grains of sand?

Wioletta Magdalena Ruszel

Department of Applied Probability
Technische Universiteit Delft, Netherlands

Abstract

The concept of self-organized criticality was introduced by Bak, Tang and Wiesenfeld in 1987 as an explanation for the existence of a certain type of noise and power-law behaviour in power spectra in particular physical systems. It captures the idea that certain physical systems can drive themselves into a critical state which shares several properties of equilibrium systems at the critical point such as power-law decay of correlations. Models of self-organized criticality provide a mechanism which can be used to explain the emergence of complexity in many natural phenomena. A toy model which displays this behaviour is the so-called sandpile model One of the most exciting applications of the sandpile dynamics is to model neuronal comunication in the brain. Here we study the abelian sandpile model on a random trees.

It was proven that for the full binary tree (and Bethe lattice) the probability that an avalanche is of size k decays as a power-law with mean-field exponent 3/2.

For the binary and binomial tree we prove exponential decay of correlations, and in a small supercritical region (i.e., where the branching process survives with positive probability) exponential decay of avalanche sizes. This shows a phase transition phenomenon between exponential decay and power law decay of avalanche sizes.

Finally we discuss our work in progress about self-organized criticality on Galton-Watson trees.

This is joint work with Antal Jarai (U Bath), Frank Redig (TU Delft) and Ellen Saada (U Paris 5).